

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

Monday 12 JUNE 2006 Afternoon 1 hour 30 minutes

Additional materials: 8 page answer booklet Graph paper List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- You are reminded of the need for clear presentation in your answers.

- 1 Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point (1, 2). [4]
- 2 (i) Expand $(1-3x)^{-2}$ in ascending powers of x, up to and including the term in x^2 . [3]
 - (ii) Find the coefficient of x^2 in the expansion of $\frac{(1+2x)^2}{(1-3x)^2}$ in ascending powers of x. [4]
- 3 (i) Express $\frac{3-2x}{x(3-x)}$ in partial fractions. [3]
 - (ii) Show that $\int_{1}^{2} \frac{3 2x}{x(3 x)} dx = 0.$ [4]
 - (iii) What does the result of part (ii) indicate about the graph of $y = \frac{3-2x}{x(3-x)}$ between x = 1 and x = 2?
- 4 The position vectors of three points A, B and C relative to an origin O are given respectively by

$$\overrightarrow{OA} = 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$

$$\overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$$

and $\overrightarrow{OC} = 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$.

- (i) Find the angle between AB and AC. [6]
- (ii) Find the area of triangle ABC. [2]
- A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2 .
 - (i) Write down a differential equation which models this situation. [2]
 - (ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]
- 6 (i) Show that the substitution $u = e^x + 1$ transforms $\int \frac{e^{2x}}{e^x + 1} dx$ to $\int \frac{u 1}{u} du$. [3]
 - (ii) Hence show that $\int_0^1 \frac{e^{2x}}{e^x + 1} dx = e 1 \ln\left(\frac{e + 1}{2}\right)$. [5]

7 Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k})$$
 and $\mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k})$,

where a is a constant.

- (i) Given that the lines are skew, find the value that *a* cannot take. [6]
- (ii) Given instead that the lines intersect, find the point of intersection. [2]

8 (i) Show that
$$\int \cos^2 6x \, dx = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$$
. [3]

(ii) Hence find the exact value of
$$\int_0^{\frac{1}{12}\pi} x \cos^2 6x \, dx.$$
 [6]

9 A curve is given parametrically by the equations

$$x = 4\cos t$$
, $y = 3\sin t$,

where $0 \le t \le \frac{1}{2}\pi$.

(i) Find
$$\frac{dy}{dx}$$
 in terms of t . [3]

(ii) Show that the equation of the tangent at the point P, where t = p, is

$$3x\cos p + 4y\sin p = 12. ag{3}$$

- (iii) The tangent at *P* meets the *x*-axis at *R* and the *y*-axis at *S*. *O* is the origin. Show that the area of triangle *ORS* is $\frac{12}{\sin 2p}$.
- (iv) Write down the least possible value of the area of triangle ORS, and give the corresponding value of p. [3]

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